Collectivity embedded in complex spectra of finite interacting Fermi systems: Nuclear example

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The mechanism of collectivity coexisting with chaos in a finite system of strongly interacting fermions is investigated. The complex spectra are represented in the basis of two-particle two-hole states describing the nuclear double-charge exchange modes in ^{48}Ca. An example of $J^{\pi}=0^{-}$ excitations shows that the residual interaction, which generically implies chaotic behavior, under certain specific and well identified conditions may create strong transitions, even much stronger than those corresponding to a pure mean-field picture. Such an effect results from correlations among the off-diagonal matrix elements, and is connected with locally reduced density of states and a local minimum in the information entropy. [S1063-651X(98)11503-9]

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I. INTRODUCTION

The concept of the random matrix theory (RMT) [1] proves very fruitful in approaching complex quantum systems and in addressing the question of how classical chaos manifests itself on the quantum level. Chaos is essentially a generic property of complex systems such as atomic nuclei [2], many electron atoms [3], molecules [4], or disordered mesoscopic systems [5] and this finds evidence in a broad applicability of RMT to describe level fluctuations [6]. Even many aspects of quantum chromodynamics are consistent with chiral RMT [7]. Similarly, however, as in most physically interesting cases where classical chaos is not just a hard billiard-type chaos, the pure RMT cannot account for the full richness of quantum phenomena connected with complexity. As an example one can mention the sign correlations [8] for parity nonconserving effects [9] in compound nuclei, even though it was the physics of compound nuclei that led Wigner [10] to postulate the Gaussian orthogonal ensemble (GOE) of random matrices as an appropriate global frame. Explicit microscopic approaches in terms of the full shell model diagonalization, either in nuclear [2,11] or atomic physics [3], show perfect agreement with GOE when looking at the local level fluctations measured in terms of the nearest neighbor spacing distribution and the Δ_3 statistics, but significant deviations take place on the level of wave functions. This originates from the two-body nature of interaction, which reduces the number of independent parameters and preserves certain correlation among the matrix elements. In order to account for this type of correlation a two-body random interaction model has been introduced [12] and its statistical properties investigated in detail [13]. Still, however, such models may not properly account for correlations that originate from the geometry of a problem and that, in some cases, may turn out to be significant.

Another characteristic connected with complexity, which is even more interesting and important from the practical point of view, is collectivity. It means a cooperation, and thus the coupling between the different degrees of freedom in order to generate a coherent signal in response to an external perturbation. Consequently, even though the real collectivity implies a highly ordered behavior it involves effects beyond the mean field — the most regular part [14] of the many-body Hamiltonian. At the same time the effects beyond the mean field are responsible for the GOE fluctuation properties. Therefore, in a sense, these two seemingly contradictory phenomena, chaos and collectivity, may have to go in parallel. Also on the classical level collectivity is a nonlinear cooperative effect that results from the coupling between different degrees of freedom.

In general, the shell model type approaches are based on diagonalization of the full many-body Hamiltonian in the basis spanned by all possible n-particle–n-hole (np-nh) configurations generated by the mean field. For practical reasons, especially when large energy intervals are involved, as, for instance, in the case of nuclear giant resonances, one truncates this hierarchy of configurations up to $2p2h$ [15]. Interestingly, due to a sufficiently large density of states relative to the strength of the residual interaction [16], local level fluctuations characteristic of GOE appear [17] to take place for the nuclear Hamiltonian acting already in the space of $2p2h$ states and this is a crucial element for an appropriate description of the giant resonance decay properties [18]. The

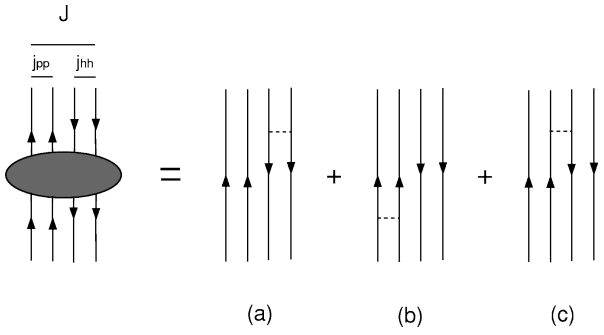

FIG. 1. Diagrammatic representation of the two-body matrix elements in the space $2p2h$ states with explicit indication of the angular momentum coupling scheme. The consecutive terms represent hole-hole, particle-particle, and particle-hole interactions, respectively.

giant resonaces are, however, excited by one-body operators that directly probe the $1p1h$ components of the nuclear wave function. The $2p2h$ states only form the background which determines a decay law. There exist, however, very interesting physical processes, represented by two-body (two-phonon) external operators, which directly couple the ground state to the space of $2p2h$ states. In view of the above mentioned local GOE fluctuations giving evidence for a significant amount of chaotic dynamics already in the $2p2h$ space, the question of a possible coherent response or collectivity under such conditions is a very intriguing one and of interest for many branches of physics.

II. MODEL

We start with the Hamiltonian that, in second quantized form, reads as

$$\hat{H}=\sum_i \epsilon_i a_i^\dagger a_i+\frac{1}{4}\sum_{ij,kl} v_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k . \tag{1}$$

The first term denotes the mean field while the second term is the residual interaction with antisymmetrized matrix elements $v_{ij,kl}$. Diagonalizing this Hamiltonian in the subspace of $2p2h$ states,

$$|2\rangle \equiv a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1}|0\rangle, \tag{2}$$

yields the eigenenergies E_n and the corresponding eigenvectors $|n\rangle=\Sigma_2 c_2^n|2\rangle$. For realistic nuclear interactions the spectral fluctuations of $\{E_n\}$ typically coincide with those of the GOE [15].

The general form of matrix elements for the two-body residual interaction $\mathcal{V}$ between 2p2h states is given by

$$\mathcal{V}_{p_1p_2h_1h_2,p_1'p_2'h_1'h_2'}=\delta_{p_1p_1'}\delta_{p_2p_2'}v_{h_1'h_2'h_1h_2}+\delta_{h_1h_1'}\delta_{h_2h_2'}v_{p_1p_2p_1'p_2'}+a(p_1,p_2)a(h_1,h_2)a(p_1',p_2')a(h_1',h_2')\delta_{p_2p_2'}\delta_{h_2h_2'}v_{p_1h_1'h_1p_1'}, \tag{3}$$

where $a(i,j)$ denotes the antisymmetrizer between i and j. The consecutive terms in this expression are responsible for hole-hole, particle-particle, and particle-hole interactions, respectively, while the remaining pair of states in each case are spectators represented by the δ_{ij} functions. These functions set a significant fraction of the matrix elements to zero, which may lead to correlations. Figure 1 illustrates the corresponding structure in diagrammatical representation. Further correlations may originate from the fact that many nonzero matrix elements relate to each other only by the geometrical factors due to the angular momentum coupling algebra.

In response to an external field $\hat{F}_\alpha$ a state

$$|F_\alpha\rangle \equiv \hat{F}_\alpha|0\rangle=\sum_n \langle n|\hat{F}_\alpha|0\rangle|n\rangle \tag{4}$$

is excited. The two-phonon operator $\hat{F}_\alpha$ can be represented as

$$\hat{F}_\alpha=[\hat{f}_\beta\otimes\hat{f}_\gamma]_\alpha, \tag{5}$$

where $\hat{f}_\beta$ and $\hat{f}_\gamma$ denote the single-phonon operators whose quantum numbers β and γ are coupled to form α. The state $|F_\alpha\rangle$ determines the strength function

$$S_{F_\alpha}(E)=\sum_n S_{F_\alpha}(n)\,\delta(E-E_n), \tag{6}$$

where

$$S_{F_\alpha}(n)=|\langle n|\hat{F}_\alpha|0\rangle|^2. \tag{7}$$

In the unperturbed basis of states $|2\rangle$ the transition strength $S_{F_\alpha}(n)$ to the state $|n\rangle$ can be expressed as

$$\begin{aligned} S_{F_\alpha}(n)&=\sum_2 |c_2^n|^2|\langle 2|\hat{F}_\alpha|0\rangle|^2 \\ &\quad+\sum_{2\neq 2'} c_2^{n*}c_{2'}^n\langle 0|\hat{F}_\alpha^\dagger|2'\rangle\langle 2|\hat{F}_\alpha|0\rangle \\ &=S_{F_\alpha}^d(n)+S_{F_\alpha}^{\mathrm{od}}(n). \end{aligned} \tag{8}$$

The second equality defines the diagonal $[S_{F_\alpha}^d(n)]$ and off-diagonal $[S_{F_\alpha}^{\mathrm{od}}(n)]$ contributions to the transition strength at energy E_n. The second component includes many more terms and it is this component that is potentially able to induce collectivity, i.e., a strong transition to energy E_n. Two elements are, however, required: (i) a state $|n\rangle$ must involve sufficiently many expansion coefficients c_2^n over the unperturbed states $|2\rangle$ that carry the strength ($\langle 2|\hat{F}_\alpha|0\rangle \neq 0$) and this is equivalent to at least local mixing, but at the same time (ii) sign correlations among these expansion coefficients should take place so that the different terms do not cancel out.

Optimal circumstances for the second condition to be fulfilled read:

$$c_2^n \sim \langle 0|\hat{F}_\alpha|2\rangle. \tag{9}$$

This may occur if the interaction matrix elements can be represented by a sum of separable terms $\mathbf{Q}^\nu$ of the multipole-multipole type:

$$\mathcal{V}_{ij,kl}=\sum_{\nu=1}^{M} Q_{ij}^\nu Q_{kl}^\nu \tag{10}$$

with $Q_{ij}^\nu \sim \langle i|\hat{f}_\nu|j\rangle$. The success of the Brown-Bolsterli schematic model [19] in indicating the mechanism of collectivity

on the $1p1h$ level points to an approximate validity of such a representation and its formal justification comes from the multipole expansion of the residual interaction. The structure of the Hamiltonian matrix in the $1p1h$ subspace is then usually dominated by few multipoles. Collectivity can then be viewed as an edge effect connected with the appearance of a dominating component in the Hamiltonian matrix and the rank M of this component is significantly lower (unity in the case of the Brown-Bolsterli model) than the size of the matrix. This rank specifies a number of the prevailing states whose expansion coefficients predominantly are functions of $\mathbf{Q}^{\nu}$. In general, on the $2p2h$ level a multipole structure of the interaction enters the corresponding matrix elements in a more complicated way. However, due to the two-body nature of the nuclear interaction, which reduces its $2p2h$ matrix elements to combinations of the ones representing the particle-particle, hole-hole and particle-hole interactions [15], the separability may become effective also on the $2p2h$ level although conditions are expected to be more restrictive. On the other hand the $2p2h$ space offers many more unperturbed transitions to form a collective state and the net effect may still appear significant.

For the quantitative discussion presented below we choose the ^{48}Ca nucleus, specify the mean-field part of the Hamiltonian (1) in terms of a local Woods-Saxon potential including the Coulomb interaction, and adopt the density-dependent zero-range interaction of Ref. [20] as a residual interaction (after correcting for a misprint in the density functional: $R_0=1.16A^{1/3}$). Since we want to inspect the higher energy region at least three mean field shells on both sides of the Fermi surface have to be used to generate the unperturbed $2p2h$ states as a basis for diagonalization of the full Hamiltonian (1). Typically, the number of such states is very large and this kind of calculation can be kept under full numerical control only for selected excitations of the lowest multipolarity. Among various nuclear excitation modes which can be considered in this context the double-charge exchange (DCX) processes are of special interest. These modes, excited in (π^+,π^-) reactions [21], involve at least two nucleons within the nucleus and give rise to a sharp peak at around 50 MeV in the forward cross section. They are thus located in the energy region of the high density of $2p2h$ states, which points to the importance of coherence effects among those states. Consequently, the present investigation may also appear helpful in studying the mechanism of DCX reactions and in separating the suggested [22] dibaryon contribution from the conventional effects [23]. For all these reasons we perform a systematic study of the DCX $J^{\pi}=0^-$ states. Our model space then develops $N=2286$ $2p2h$ states. There are still several possibilities of exciting such a double-phonon mode represented by the operator $\hat{F}_{\alpha}$ out of the two single phonons $\hat{f}_{\beta}$ and $\hat{f}_{\gamma}$ of opposite parity. For definiteness we choose

$$\hat{f}_{\beta}=rY_1\tau_- \tag{11}$$

and

$$\hat{f}_{\gamma}=r^2[Y_2\otimes\sigma]_{1^+}\tau_- . \tag{12}$$

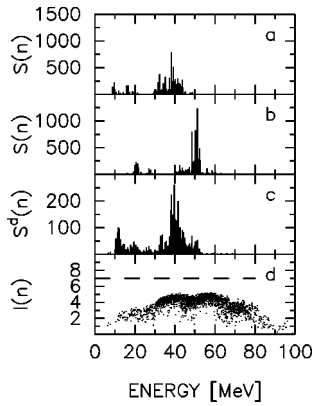

FIG. 2. (a) The unperturbed transition-strength distribution in ^{48}Ca for the $J^{\pi}=0^-$ DCX excitation involving the single-phonon dipole and $2\hbar\omega$ spin-quadrupole modes. (b) The same as (a) but after including the residual interaction. (c) $S^d_{F_\alpha}$ components of the transition strength as defined by Eq. (8) (notice different scale). (d) The information entropy of the states $|n\rangle$ in the unperturbed basis. The dashed line indicates the GOE limit $[\ln(0.48N)]$.

The first of these operators corresponds to the $1\hbar\omega$ dipole and the second to $2\hbar\omega$ spin-quadrupole excitation. The resulting two-phonon mode thus operates on a level of $3\hbar\omega$ excitations. Formulas needed to express the angular momentum coupled form of the above one- and two-body operators can be found, for instance, in the Appendix of Ref. [24].

III. RESULTS AND DISCUSSION

The results of calculations are presented in Fig. 2. As one can see, including the residual interaction [part (b)] induces a strong transition at 51.1 MeV. This transition is stronger by almost a factor of 2 than any of the unperturbed [part (a)] transitions even though it is shifted to a significantly higher ($\sim$ 10 MeV) energy. This is also a very collective transition. About 95% of the corresponding strength originates from $S^{\rm od}_{F_\alpha}(n)$, as comparison between parts (b) and (c) of Fig. 2 indicates. This whole effect is due to particle-hole type matrix elements [Fig. 1(c)]. Discarding diagrams (a) and (b) produces no significant difference. The degree of mixing can be quantified, for instance, in terms of the information entropy [25]

$$I(n)=-\Sigma_i p_i \ln p_i , \quad p_i=|c^n_i|^2 \tag{13}$$

of an eigenvector $|n\rangle$ in the basis [part (d)]. Interestingly, the system finds preferential conditions for creating the most collective state in the energy region of local minimum in $I(n)$. Our following discussion is supposed to shed more light on this issue.

As shown in Fig. 3(a) our Hamiltonian matrix displays a bandlike structure with spots of the significant matrix elements inside. This together with a nonuniform energy distribution $\rho_u(E)$ of the unperturbed $2p2h$ states [Fig. 3(b)], which is a trace of the shell structure of the single-particle states, characteristic of many other mesoscopic systems [26], sizably suppresses the range of mixing and locally supports conditions for the edge effect to occur in the energy region of the minimum in $\rho(E)$. A comparison with Fig. 2(b) shows that the collective state is located at about this region. More-

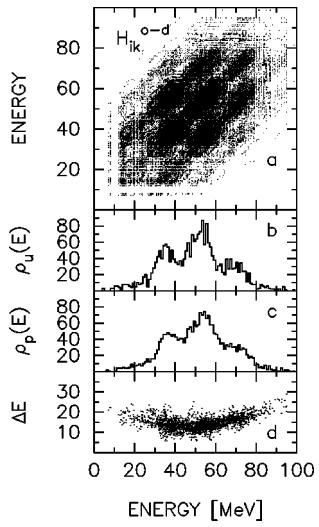

FIG. 3. (a) Structure of the Hamiltonian matrix for the $J^\pi = 0^-$ DCX states. The states are here labeled by energies, ordered in ascending order and the matrix elements $H_{ik} \geq 0.1$ are indicated by the dots. (b) Density of the unperturbed $2p2h$ states. (c) Density of states after the diagonalization. (d) Energy range of interaction between the unperturbed states.

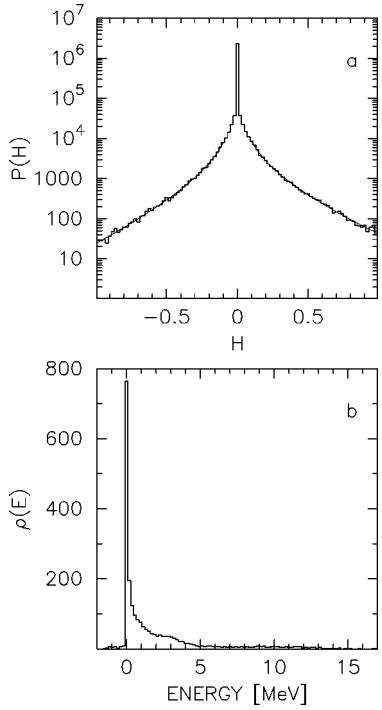

FIG. 4. (a) Distribution of off-diagonal matrix elements between the $J^\pi = 0^-$ DCX states (histogram). The solid lines indicate fit in terms of Eq. (15) with the resulting parameters: $a = 676$, $b = -1.21$, $c = 0.69$ (left) and $a = 692$, $b = -1.22$, $c = 0.81$ (right). (b) Density of states corresponding to the residual interaction part of the Hamitonian (1).

over, the minimum survives diagonalization [$\rho_p(E)$ in Fig. 2(c)] and all the above features are consistent with the effective band range [27]

$$(\Delta E_i)^2 = \sum_j (H_{ii} - H_{jj})^2 H_{ij}^2 \Big/ \sum_{i \neq j} H_{ij}^2 \qquad (14)$$

shown in Fig. 2(d).

Further quantification of the character of mixing between the unperturbed states is documented in Fig. 4. The distribution $P(H)$ of off-diagonal matrix elements (a) is not Gaussian but of the following type:

$$P(H) = a|H|^b \exp(-|H|/c). \qquad (15)$$

This indicates the presence of the dominating multipole-multipole components in the interaction [2,3]. An interesting feature is the asymmetry between the positive and negative valued matrix elements [see parameters in the caption to Fig. 4(a)]. The positive matrix elements are more abundant, which expresses further correlations among them and the fact that the interaction is predominantly repulsive for the mode considered. Significant reduction of dimensionality is also indicated by the distribution of eigenvalues of the residual interaction. As shown in part (b) of Fig. 4 the majority of these eigenvalues is concentrated around zero and thus constitute approximate zero modes of that part of the Hamiltonian. We also would like to note at this point, without showing the results explicitly, that similar analysis on the $1p1h$ level using appropriately larger model spaces (for better statistics) shows an even larger fraction of such zero modes. This is due to the fact that in the $1p1h$ space the multipole-multipole structure of the interaction manifests itself in a more transparent way.

Appearance of a strong transition at certain energy E_n means that the structure of the Hamiltonian matrix of the residual interaction, at least locally at around that particular energy, is governed by a component of the type as specified by Eq. (10) with a small number of terms ($M \ll N$) including, of course, the ones that coincide with an external field. In a pure case a structure like this causes an energy gap between the collective state and the remaining states. In the present case, of its only local nature, one expects a local minimum in the density of states in the vicinity of the collective state. Indeed, as can be seen by a careful inspection of Fig. 2(b) versus Fig. 3(c) any stronger transition is located in such a minimum whose range typically extends over an energy interval of the order of 0.5 MeV. Even relatively weak transitions are asigned their own minima. Moreover, as we have verified in certain selected cases, many other minima in the density of states that are not occupied by the above specified transitions turn out to be filled in by the DCX $J^\pi = 0^-$ transitions connected with other combinations of two one-phonon operators (for instance, $\hat{f}_\beta = r^2 Y_2 \tau_-$ and $\hat{f}_\gamma = r[Y_1 \otimes \sigma]_{2^-} \tau_-$).

A reduction of the rank (real dimensionality) of the Hamiltonian matrix evidenced above is also consistent with the observed minimum in the information entropy [Fig. 2(d)]. Simply, in the relevant energy region there are fewer free parameters and this sets additional constraints on the degree of mixing and thus on the amount of chaos. As a chaos related characteristics we take the spectral rigidity measured in terms of the Δ_3 statistics [1]. We find this measure more appropriate for studying various local subtleties of mixing than the nearest neighbor spacing (NNS) distribution because for a smaller number of states the latter sooner becomes contaminated by strong fluctuations. Indeed, the spec-

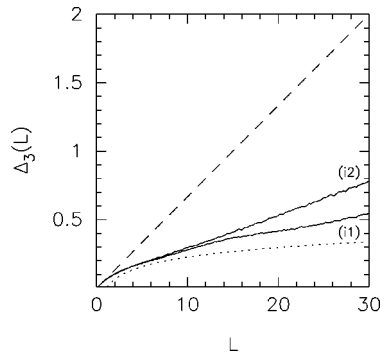

FIG. 5. Spectral rigidity $\Delta_3(L)$ for eigenvalues from the two 400-state long intervals: $n=351-750$ (i1) and $n=751-1150$ (i2). The long-dashed line corresponds to Poisson level distribution and the short-dashed line to GOE.

tral rigidity (Fig. 5) detects differences in the level repulsion inside the string of eigenvalues (i1) covering the first maximum in $\rho_{p(u)}(E)$ (35.2–44.5 MeV, 400 states starting from $n=351$ up to $n=750$) and the one (i2) covering the minimum and thus including the collective state (44.5–52.1 MeV, 400 states from $n=751$ to $n=1150$). The deviation from GOE is more significant in i2, which, similarly as $I(n)$, signals a more regular dynamics in the vicinity of the collective state ($n=1089$).

Conditions corresponding to the actual Hamiltonian are not the most optimal ones from the point of view of the collectivity of our $J^\pi=0^-$ DCX excitation. By multiplying the residual interaction by a factor of $g=0.7$ we obtain a picture as shown in Fig. 6. Now the transition located at 48.6

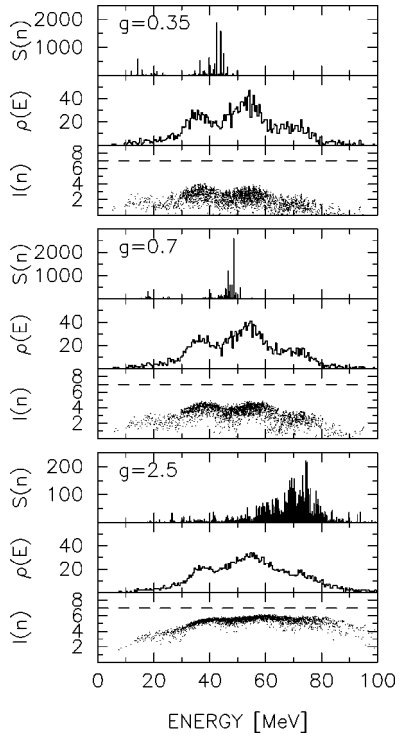

FIG. 6. Transition-strength distribution, density of states and the information entropy for the same excitation as in Fig. 1 but the residual interaction is now multiplied by a factor of $g=0.35$ (upper part), $g=0.7$ (middle part), and $g=2.5$ (lower part), respectively.

MeV is another factor of 2 stronger than before and, again, all significant transitions are situated in the local minima of $\rho(E)$ and in the overall minimum of the information entropy. A too severe decrease of g will eventually bring all the transitions to their mean-field values. This transition is, however, not just linear. Here we seem to be facing a competition of the two elements. One is the residual interaction that must be sufficiently strong to correlate many states but the other one is a condition for the edge effect to occur. As a result, even for $g=0.35$ we still obtain very strong transitions, apparently due to the fact that the interaction strength is such that the unperturbed transitions are moved just to the absolute minimum in $\rho(E)$.

The range of values of a multiplication factor that produces this kind of picture is rather narrow and this feature of collectivity resembles a classical phenomenon of the stochastic resonance [28]. It is relatively easy to completely destroy such strong transitions. By multiplying the residual interaction by a factor of $g=2.5$ (which is equivalent to increasing the density of states) the strength distribution displays a form as shown in the lowest panel of Fig. 6. This strength remains largely localized in energy but the distribution of the corresponding $S_F(n)$ (Fig. 6) does not deviate much from the Porter-Thomas (PT) distribution [29] $P(s)=(2\pi s)^{-1/2}\exp(-s/2)$ characteristic of GOE, even though correlations among the matrix elements are the same as before. Interestingly, even here the larger transitions are located in their own small minima in $\rho(E)$. Further increase of the multiplication factor may again produce some transitions that are more collective than those allowed by PT. In particular, starting from values ~ 4 some new strong collective transitions appear at the upper edge of the whole spectrum.

To illustrate statistics of the transition strength versus PT distribution we use a measure introduced in Ref. [30]. Consequently, for all the cases considered above we calculate the total number N of transitions of magnitude smaller than a given threshold value $S_{\rm th}$, as a function of $S_{\rm th}$. Since the number of large components relative to the small ones is of primary interest in the present study and in order to set the same scale when comparing different cases we, in addition, in each case independently, divide all the transitions $S(n)$ by the corresponding maximum value of $S(n)$. After that $S_{\rm max}(n)=1$ in each case. Consistently, the RMT limit of this measure is then drawn from the cummulative PT distribution and this limit is indicated by the solid line in Fig. 7. In the log-log scale this limit develops a long straight line segment with the slope of 0.5, which reflects the dominant role of the preexponential factor ($s^{-1/2}$) in PT distribution at smaller transitions. As can be seen from this figure, the $g=2.5$ case is very close to this limit. But, interestingly, even $g=0.35$ tends to the same slope when probing the region of small transitions, which means that such transitions are consistent with GOE. Only the unperturbed case is distinct in this sense.

Finally, Fig. 8 shows the transition strength distribution of the ''constituent'' single-phonon modes specified by Eqs. (11) and (12), respectively, in their own $1p1h$ sectors and the same model space of single-particle states is used. There are 28 (single) charge exchange $J^\pi=1^-$ and 25 $J^\pi=1^+$ $1p1h$ states in this space. As before, the g factors reflect the strength of the residual interaction relative to the original

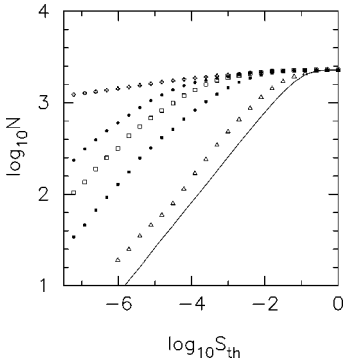

FIG. 7. The total number N of transitions of given strength (properly rescaled, see text) below a threshold value S_{th}. The open crosses refer to the unperturbed case ($g=0$), thick dots to $g=0.35$, open squares to $g=0.7$, filled squares to $g=1$, and open triangles to $g=2.5$. The solid line represents the same quantity determined from a Porter-Thomas distribution.

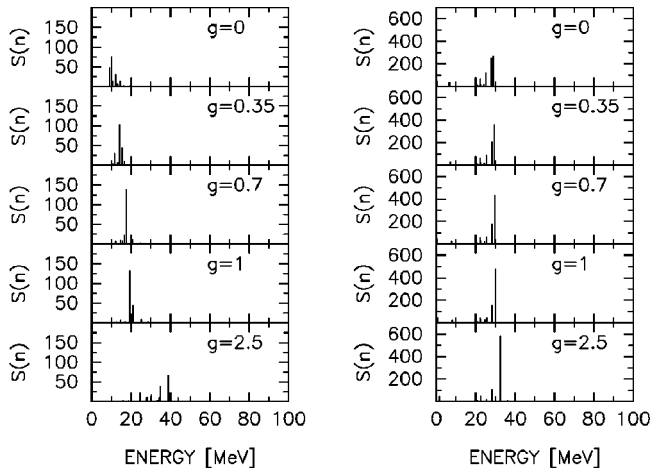

FIG. 8. Transition-strength distribution corresponding to single-phonon operators specified by Eq. (11) (left) and by Eq. (12) (right), calculated in the space of $1p1h$ states for various residual interaction multiplication factors g.

one. The results collected in this figure provide further evidence that collectivity observed on the $2p2h$ level is not accidental. It can always be traced back to collectivity of the corresponding single-phonon modes in their own subspaces. Consistently with our previous discussion, this correspondence cannot, however, be expressed simply in terms of proportionality. For instance, the single-phonon transitions for $g=0.35$ are significantly weaker than for $g=1$ while the opposite applies to the resulting two-phonon mode. Recent study of Ref. [31], even though based on a much simpler model, also shows that characteristics of the two-phonon mode (double giant dipole) are much more sensitive to the detailed form of the Hamiltonian than those of the corresponding single-phonon modes.

Taken together, a real collectivity, by which we mean a transition stronger than those generated by the mean field, is a very subtle effect and is not a generic property of the complex spectra. Its appearance, as it happens for one of the components of the $J^{\pi}=0^{-}$ DCX excitations considered here, involves several elements like correlations among the matrix elements, nonuniformities in the distribution of states and a proper matching of the interaction strength to an initial (unperturbed) location of the transition strength relative to the scale of nonuniformities in the distribution of states. If present, a collective state is then located in the region of more regular dynamics characterized by lower information entropy, more sizable deviations from GOE of the level fluctuations and local minima in the density of states. This later effect can thus potentially be used in experimental studies as an extra criterion to detect collectivity. We also would like to point out that these aspects of collectivity parallel an analogous property hypothesized for living organisms [32] and stating that collectivity is a phenomenon occurring at the border between chaos and regularity.

ACKNOWLEDGMENTS

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